Lab 4
Frequency Translation
EE103L

1.1 Purpose:

This lab continues our investigation with experimental signals in both the time and frequency domains using the laboratory oscilloscope and spectrum analyzer for observation and analysis. We are interested in studying what happens when two sinusoids are passed through a particular kind of non-linear circuit that multiplies them together. This will enable us to experimentally illustrate the *frequency shifting theorem* and observe how it is used in communications systems, where the process is called *frequency mixing*. Depending on the application, this may be termed *modulation, mixing* or *heterodyning*.

1.2 Background:

Linear time-invariant networks or circuits obey the defining relationship of superposition. This means that if we apply a single sinusoid to such a circuit, the response can change the input signal *only in amplitude and phase*. Thus, in particular, it can contain no new frequencies that weren't already present in the input. Hence, if a group of sinusoids are applied by adding them together, each one has no effect on the others and all appear at the output as if it were applied to the network alone. If this group also happens to be a Fourier series of harmonic signals where their instantaneous sum (or superposition) *represents* the actual complicated non-sinusoidal time signal passing through the network, each can only be modified in amplitude and time delay (phase). We can say this kind of network is defined more precisely by the following two properties:

1. The response to an instantaneous sum of excitations applied as a single signal is equal to the instantaneous sum of the responses to each of these excitations acting in isolation.

2. The relationship between the input and output does not vary with time; it is time-invariant or stationary.

Consider the practical case of a linear audio amplifier. Whenever the volume control is changed, the circuit is linear before and after but not *during the gain change*. While the gain is changing, it is no longer time-invariant because the amplitude relationship between the input and output is varying over time and, during this interval, the amplifier *does not obey superposition*. even though we may describe the amplifier alone as being linear and time-variant while the volume control is changing.

Now suppose the volume control is left alone, but instead the amplifier itself has a non-linear gain curve, one whose gain varies with the amplitude of the input. This circuit also *does not obey superposition*. This is the usual sense when we say a circuit or network is "non-linear": we mean it is "non-linear and time-invariant". The second sense of non-linearity is less obvious when a circuit happens to be "linear and time-variant". Neither obeys superposition. You should reserve the unqualified use of "linear" only when it also implies time-invariance. Generally, when considering a circuit's linearity, the litmus test is always whether it obeys superposition.

Non-linear circuits do not obey superposition since new frequencies not present at the input are generated that appear at the output. A non-linear circuit also has no impulse response $h(t)$ (why?). Applying a single frequency (or "tone") will always result in the creation of new frequencies harmonically related to the input. For example, one measure of an audio amplifier's linearity is called *total harmonic distortion* or THD. To measure it, a single very low-distortion tone is applied to the input. Since the output contains the original tone's frequency plus spurious harmonics generated by the amplifier's non-linearity, a special AC voltmeter having an internal filter tuned to remove only the fundamental frequency is used to measure the RMS sum of all remaining harmonics. This result is then divided by the RMS voltage of the input sinusoid and expressed as a percentage. If you look on hi-fi equipment specifications, you will see this as a commonly cited parameter of merit. Special lab instruments, called harmonic distortion analyzers designed to operate in the audio region (30 Hz to 20kHz) are available to make these measurements.

1.3 Frequency Translation using an Amplitude Modulator:

The multiplier we will observe and study in this lab is an active 3-port network having two inputs and one output. It is sometimes described as *linear and time-variant*, and therefore, is essentially *non-linear* in its input-output relationship. Two *new* frequencies will necessarily be present in the output: the sum and difference of the inputs. Unlike other non-linear circuits these signals aren't considered spurious; they're specifically *wanted* and quite useful! Why? In a nutshell, without this device, we would be unable to build radio transmitters or receivers. Your familiar cell phone wouldn't be possible. The principle of operation was invented by Edwin Armstrong in 1918 and termed *heterodyning*[1]. The basic idea is to phase-coherently move spectral energy to another range of frequencies without losing any information (that's what "phase-coherent" means). For example, classic commercial AM stations reside between about 500 kHz and 1600 kHz. Circuitry used to extract the audio being carried by any particular station's assigned *carrier* frequency is designed to operate at only one frequency: 455 kHz. So a frequency mixer is used to translate AM receiver signals down to this one single frequency. FM stations, operating between 88 and 108 MHz do this the same way. A mixer is used to translate any wanted station down to 10.7 MHz where the FM audio can be extracted.

A converse example is the engineering problem of getting signals in the audio range so they can be transmitted by an AM radio station. The technique is to mix the audio with a fixed, highly stable sinusoidal oscillation precisely on the AM station's assigned frequency. This process is called *modulation* and results in new sum and difference signals precisely centered on the station's carrier frequency. The carrier is aptly named, for it "carries" the two new frequencies as side-bands symmetrically spaced on either side of it. This technique is called amplitude modulation or just AM for short. You will be able to observe this using the scope and spectrum analyzer.

[1] See Wikipedia's entertaining and informative article at http://en.wikipedia.org/wiki/Superheterodyne_receiver.

There are many circuit variations capable of mixing action. The one we will be using is a classic variable gain amplifier (VGA). This will be more fully explained in class. For now, we'll review the basic mathematical relationships:

Our mixer or frequency multiplier accepts two inputs, $v_c(t)$ and $v_m(t)$. The subscripts refer to the carrier, "c", and the sinusoid to be carried as the message, "m". We will denote the frequency translated output as the AM with subscript "am". For AM, the carrier is always many times greater than the modulation. For example, consider an AM radio station on 1000 kHz carrying a 5 kHz modulated tone. For every cycle of the 1 kHz sine wave, 200 cycles of the carrier occur. Mathematically, we can express the relationship as follows:

[1] $v_m(t) = 1 + \cos(2\pi f_m t)$, the "message" sinusoid.

[2] $v_c(t) = \cos(2\pi f_c t)$, the carrier sinusoid.

[3] $v_{am}(t) = v_c(t)v_m(t) = \cos(2\pi f_c t)\left[1 + \cos(2\pi f_m t)\right]$, the full AM carrier.

You can see that eqn. (3) clearly expresses the non-linear multiplier relationship, where the carrier is multiplied, or modulated, by the message. Using trigonometric identities, this quickly reveals that two *new* sum and difference frequencies, frequencies that were not in either input, are immediately apparent:

[4] $v_{am}(t) = \cos(2\pi f_c t) + \frac{1}{2}\left[\cos 2\pi\left(f_c - f_m\right)t + \cos 2\pi(f_c + f_m)\right]$

Eqn. (4) has three terms, each representing a different frequency component from the mixer's output. The first is the carrier frequency. This was already present in the input and appears because the message was biased up to keep it positive, *i.e.*, 1 [V] was added to the carrier cosine so it varies between 0 and 2 [V] instead of -1 and 1 [V]. It should be evident that this DC offset resulted in the carrier being present in the output.

The second and third terms are the two new frequencies symmetrically spaced about the carrier. $f_c - f_m$ is called the *lower side-band* (LSB), while $f_c + f_m$ is the *upper side-band* (USB). Notice that as f_m is *increased* the side-bands *move away* from the carrier by equal amounts proportional to the message frequency. If the message were speech or music and extended from, say, 50 Hz to 7.5 kHz in the audio range, they would occupy a band of frequencies in this same range on either side of the carrier. This is exactly how classic commercial AM radio works. The transmitted signal contains a carrier and two full side-bands, so it is designated as DSB-FC, "double side-band full-carrier."

A more compact form of the relations described in eqn. (4) can be derived from the *frequency shifting theorem*:

[5] $g_c(t)e^{j\omega_m} \leftrightarrow F(\omega - \omega_m)$.

Since the complex exponential is not a real function we can physically generate in the lab, it is replaced with a sinusoid, giving a more practical form:

[6] $$g_c(t)\cos(\omega_m t) = \frac{1}{2}\left[g(t)e^{j\omega_m t} + g(t)e^{-j\omega_m t} \right].$$

Where it follows that

[7] $$g_c(t)\cos(\omega_m t) \leftrightarrow \frac{1}{2}\left[G(\omega_c - \omega_m) + G(\omega_c + \omega_m)\right]$$

Adding the 1[V] offset from eqn. (1) results in an equivalent form:

[8] $$g_c(t)\left[1+\cos(\omega_m t)\right] \leftrightarrow G(\omega_c) + \frac{1}{2}\left[G(\omega_c - \omega_m) + G(\omega_c + \omega_m)\right]$$

Each of the three Fourier transform terms on the right side results in impulses at the three radian frequencies, ω_c, $\omega_c - \omega_m$ and $\omega_c + \omega_m$ respectively. Remember the last two are a consequence of the non-linear operation of multiplication.

1.4 Assignment

Before getting started on the lab, necessary practical topics will be thoroughly discussed by lab staff along with two supporting handouts:

- A description of the actual circuit along with how to set it up and use it.
- Graphs of typical waveforms you can expect to see.

You are to observe and verify basic relationships between the time-domain signal seen on the scope and frequency domain signals seen on the spectrum analyzer.

To help you get started, begin with a carrier frequency of 300 kHz and modulation frequency of 1 kHz. Using the scope set the modulation for 100% by observing the carrier envelope swinging to 0. 100% corresponds to the maximum amplitude of the message before further non-linear distortion begins to appear along with the wanted side-bands.

> ***Note:*** Keep ac voltages as low as possible to avoid overdriving the spectrum analyzer. Use of a 10:1 probe on the analyzer which has a 50 Ohm input is very useful to drop signals significantly and thereby allow the scope to view clean higher amplitude signals. Use this if it's available.

Spectrum analyzer settings:

- Span = 10 kHz centered on 300 kHz
- Resolution Bandwidth (RBW) = 100 Hz

Once you get the hang of things, vary the carrier and modulation frequencies to observe what happens in both the time and frequency domains. The idea here is to gain some intuition and insight that should help you understand the math more clearly. Note that you should keep the carrier frequencies between 300 and 500 kHz and modulation frequencies between 1 kHz and 20 kHz. Make good sketches of what you see on the scope and spectrum analyzer and turn in at least three well-drawn sketches describing their meaning.

Be sure to also verify by observation of the spectrum analyzer and by direct calculation that power in the side-bands at 100% modulation is half that of the carrier:

[8] $$P_{Sidebands} = \frac{1}{2} P_{carrier}$$

Submit your measurements, calculations and supporting waveform sketches.

Have fun!

EE 103L ①

Re: ELECTRONIC COMM. SYSTEMS
By WAYNE TOMASI, Prentice Hall 1988

Coefficient of Modulation and Percent Modulation

Coefficient of modulation is a term that describes the amount of amplitude change (modulation) in an AM envelope. *Percent modulation* is simply the coefficient of modulation stated as a percentage. More specifically, percent modulation gives the percentage change in the amplitude of the output wave when the carrier is acted on by a modulating signal. Mathematically, modulation coefficient is

$$m = \frac{E_m}{E_c} \tag{3-1a}$$

where

m = modulation coefficient
E_m = peak change in the amplitude of the output wave
E_c = peak amplitude of the unmodulated carrier

Equation 3-1a can be rearranged to solve for E_m or E_c:

$$E_m = mE_c \qquad \text{and} \qquad E_c = \frac{E_m}{m} \tag{3-1b}$$

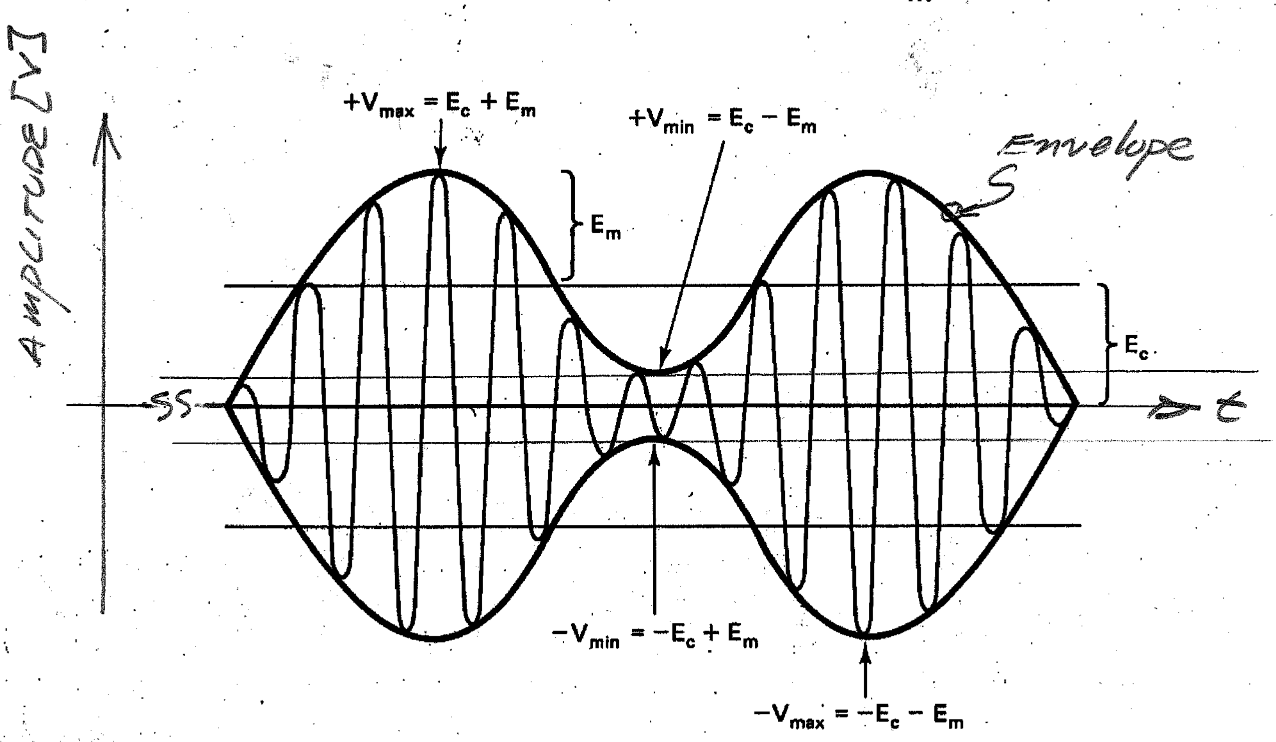

FIGURE 3-5 Modulation coefficient, percent modulation, E_m, and E_c.

TIME-DOMAIN WAVEFORMS

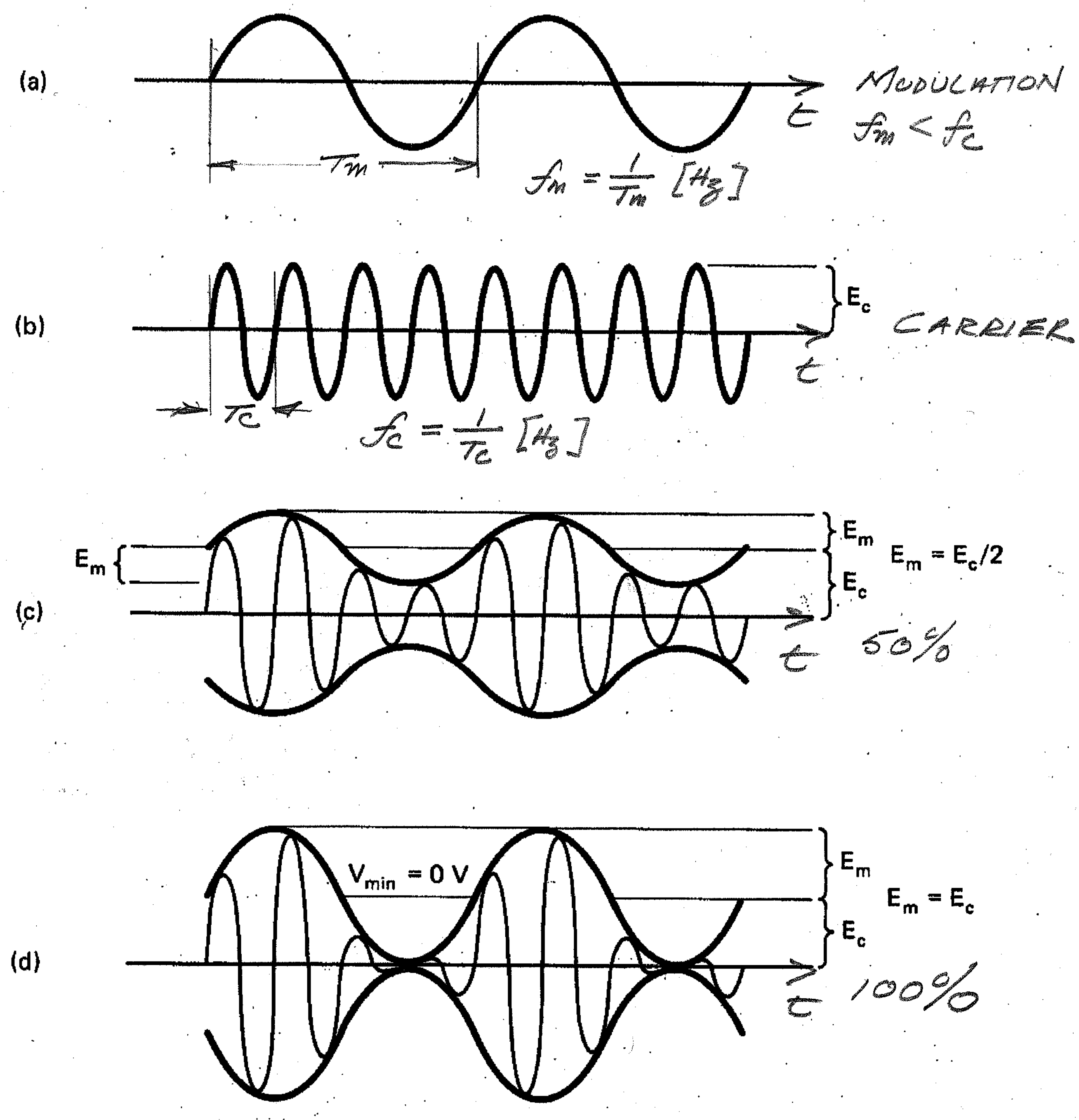

FIGURE 3-6 Percent modulation of an AM DSBFC envelope: (a) modulating signal; (b) unmodulated carrier; (c) 50% modulated wave; (d) 100% modulated wave.

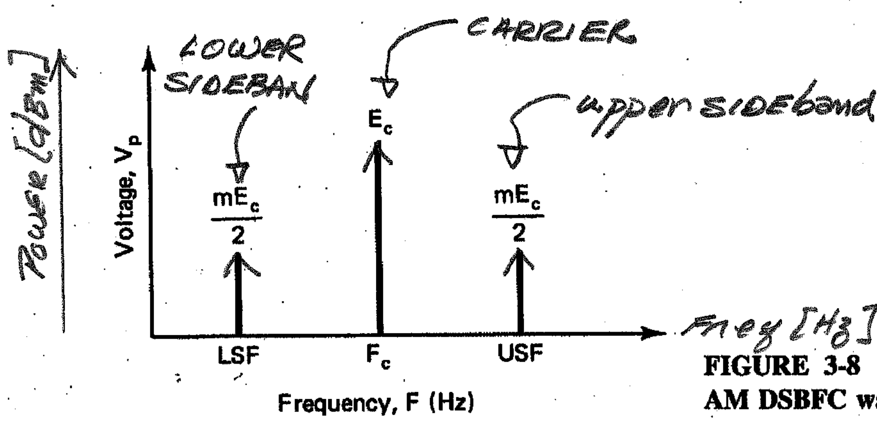

FIGURE 3-8 Voltage spectrum for an AM DSBFC wave.

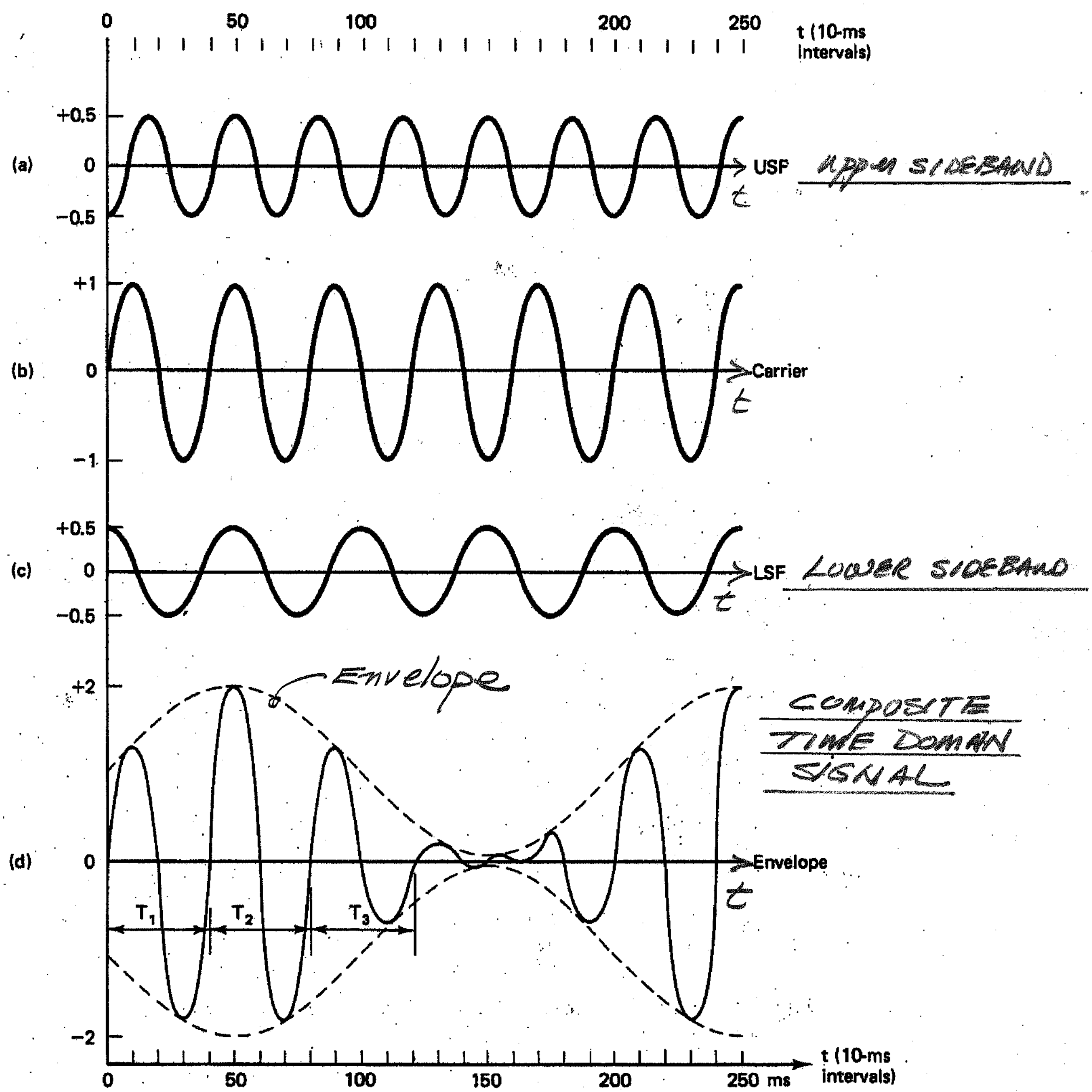

FIGURE 3-9 Generation of an AM DSBFC envelope shown in the time domain: (a) $-\frac{1}{2}\cos 2\pi 30t$; (b) $\sin 2\pi 25t$; (c) $+\frac{1}{2}\cos 2\pi 20t$; (d) summation of (a), (b), and (c).

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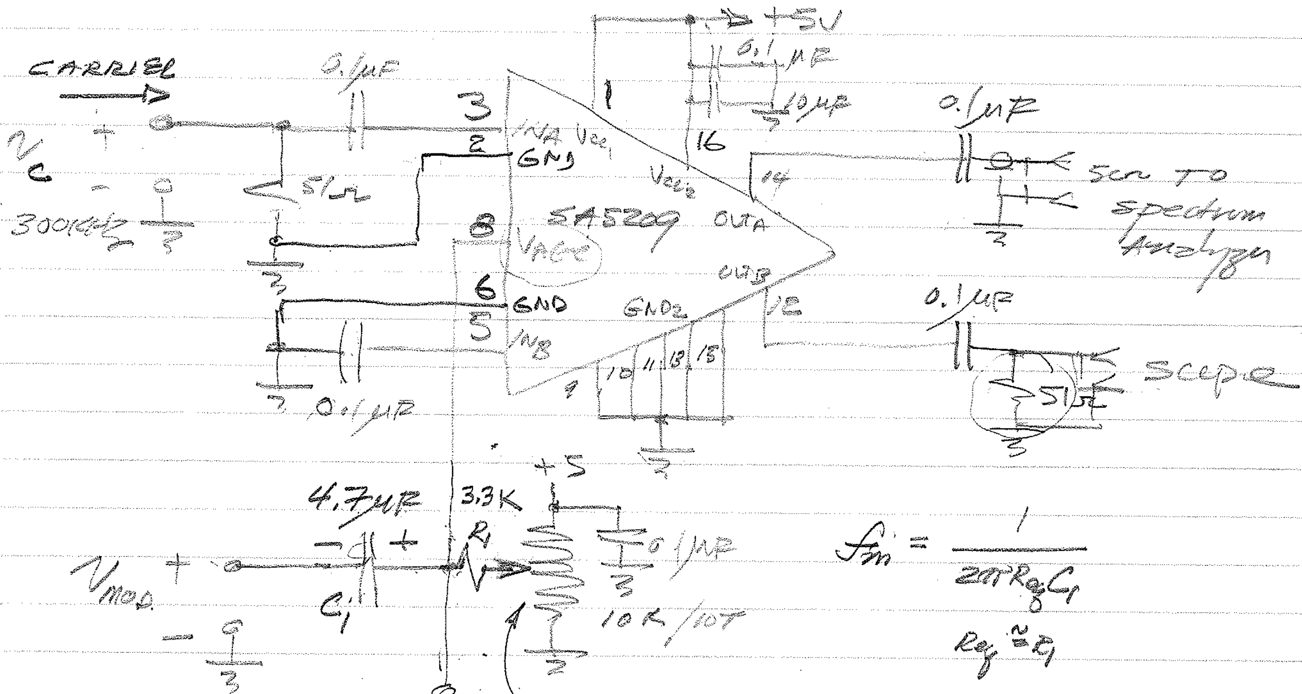
AM MODULATOR

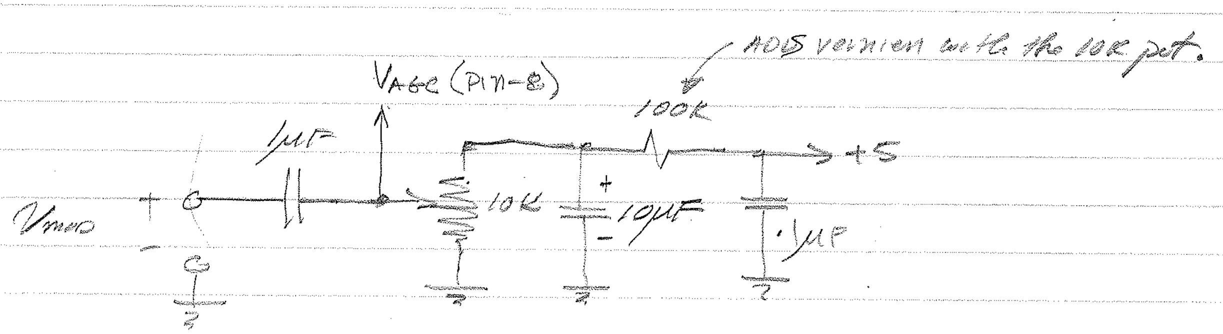
1	V_{CC_1}	V_{CC_2}	16
2	GND_1	GND_2	15
3	IN_A	OUT_A	14
4	GND_1	GND_2	13
5	IN_B	OUT_B	12
6	GND_1	GND_2	11
7	V_{BG}	GND_2	10
8	V_{AGC}	GND_2	9

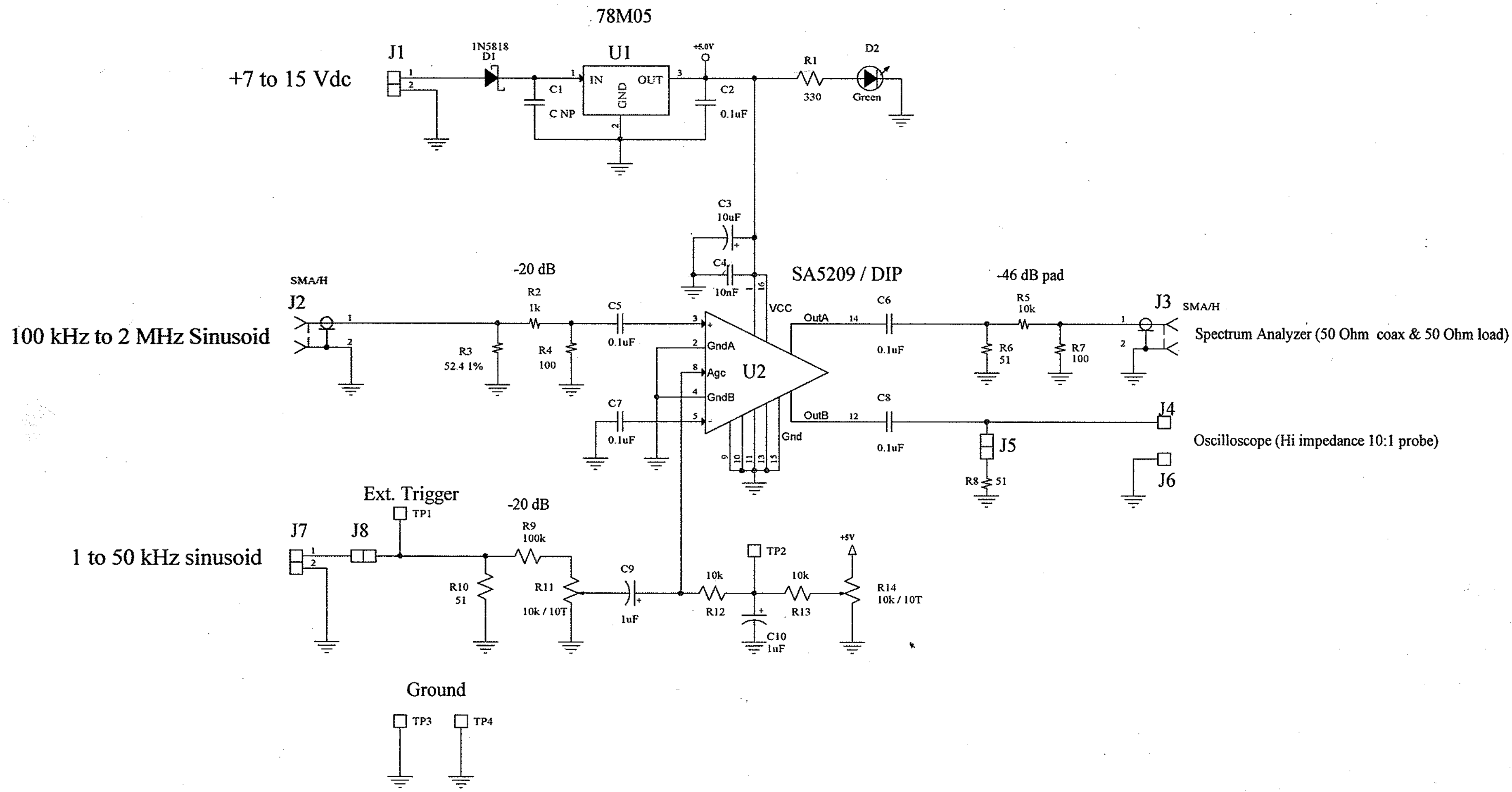


$$f_m = \frac{1}{2\pi R_{eq} C_1}$$

$R_{eq} \cong R_1$

T.P. (set for 1.0 [V/V] Gain) ≈ 0.1 → 0.2 V_{DC}





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