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Hardware Laboratory 1 Introduction to Test Equipment and Time and Frequency Domains

EE103L
System and Signals Laboratory

DESCRIPTION AND OBJECTIVE

Electric signals can be expressed equivalently in either the time or frequency domains. This laboratory introduces basic test equipment needed to study the strong relationship between the two domains by observing experimental waveforms of the same signal in both. Our objective is to develop some insight in mentally visualizing waveforms in either of these two domains, and to experimentally confirm relevant underling linear system theory.

We will be working with four basic pieces of hardware measurement test equipment used with this series of hardware labs. These include:

Oscillosope, (informally “scope” or “O’scope”) a time-domain instrument for graphically viewing voltage signals versus time on a rectangular X-Y grid, where the vertical Y-axis represents voltage and the X-axis the linear elapse of time. Reasonably accurate measurements of voltage magnitude and time are possible with this instrument, but its most prevalent use is not for precise measurements but to subjectively view and assess a signal’s *waveform* – basically “what it looks like”. We can often tell a great deal about a signal’s time and frequency-domain properties just by viewing its wave shape. This involves subjective features, like the degree of “rounding” on a square-wave, or what its “rise” or “fall” times look like, or a sine wave distorted by “flattening:” or “sharpening”. None of these lend themselves easily to simple precise quantitative measurements.

Digital Voltmeter, (informally “dvm” or “vm”), a time-domain averaging instrument used to precisely measure DC voltages and AC rms voltages and currents sbeyond what the scope is capable of. The dvm will is also used to measure the value of resistors.

Signal Generator, a time-domain instrument, also known as an “arbitrary waveform generator” (“arb”) or “function generator”. This instrument is used to generate precise voltage signals representing classic common waveforms, including *sine*, *triangle*, *square*, and programmable non 50% duty cycle waveforms.

Spectrum Analyzer, a frequency-domain instrument that works much like the o’scope. It is used for graphically viewing power signals versus Hertzian frequency, where the vertical Y-axis represents logarithmic variations in power and the X-axis represents the linear sweep of frequency [Hz.]. Thus, it displays power at various frequencies. At this point, think of it much like a radio receiver with a precise tuning dial where you can view the presence of energy at particular tuned frequencies and, instead of listening to them, you see them. This instrument will probably be new to most of you and is undoubtedly the most fun to use! As you will discover, it’s quite easy to learn, but will require your close attention to the basic concepts at the

beginning. Together with the scope, it enables us to view a signal's representation in both the time and frequency domains simultaneously.
Because we only have four of these very expensive spectrum analyzers in the undergraduate laboratory, they must be used on a shared basis with students typically working in teams.

2.1 Fourier series

As you know from lecture, a periodic time domain signal can be represented in the frequency domain by a Fourier series. Fourier's basic idea was that time domain functions periodic in 2π radians can be represented as an infinite sum of harmonically related frequency domain sinusoids having different amplitudes and phases.

2.2 Power in the harmonics

As should already know, power is conserved between the time and frequency domains. We will experimentally verify this fundamental physical principal. To do so, you must carefully pay attention to the units associated with each piece of test equipment. The function generator is calibrated to produce a default output in peak-to-peakVolts (V_{pp}) when driving a 50 Ohm load. However, when measuring an AC voltage, the DVM is calibrated to always display the equivalent DC voltage corresponding to the average power that would be dissipated were it connected to a resistance; this voltage of course is V_{rms}. For a sinusoid this is known from previous coursework (EE101) to be $\frac{V_p}{\sqrt{2}}$ or approximately $0.7071\, V_P$ (where V_p is the peak voltage). Waveforms other than sinewaves you will have to look up or determine by calculation.

The spectrum analyzer is also calibrated much like the DVM, except it always displays average power across its actual 50 Ohm input resistance. Note too that this resistance is in series with a DC blocking capacitor to prevent damage to the input amplifier stage. Normally the displayed wwould be in Watts, but because the instrument can measure power over about 10 orders of magnitude; yes that's right, this is about $10^{10}\left[\frac{W}{W}\right]$! , a linear display is impossible. So, it is always expressed in decibels. Since decibels are based on logarithmic power ratios, the reference power is 1 milliwatt, [mW], defined as: $dBm = 10\log_{10}\frac{p_{50\Omega}}{1[mW]}$. Thus, 0 dBm is 1[mW], 10 dBm is 10 [mW], 30 dBm is 1[W] and 50 dBm is 100 [W], -30dBm is 1[μW], etc. You should remember from physics that power is the time rate of change of energy, $p(t) = \frac{dw}{dt}$, where power is in Watts and energy, *w*, is in Joules. Electrical power varies with the square of voltage or current: $p(t) = \frac{v(t)^2}{R} = i(t)^2 R$. Given a signal x(t) that could be Volts or Amperes, we often generalize this idea to simply mean $x(t)^2$ as the instantaneous power at time *t*. Carefully note this is *not* dimensionally correct, but we do so with the usual agreement that we let $R = 1\Omega$. You will often see this in mathematical treatments for the sake of brevity.

Now, carefully note the signals generated in this lab by our test equipment are assumed to have existed for eternity so, theoretically at least, they are assumed to have infinite energy but constant power when averaged over any periodic cycle. Such eternal signals are classified as *power signals*. All periodic signals having a Fourier series representation are power signals, and this includes any such signal we can experimentally generate in the lab.

We now get to the point of this section. A periodic power signal expressed in either the time or frequency domains must have the same power. Thus, if we display a single sinusoid on the oscilloscope and observe that same signal as a single line on the spectrum analyzer, they must display the same average power. Thus, if the O'scope displays this single sinusoid having a peak voltage of v_{pk} and the spectrum analyzer some power in dBm, p_{dBm} then, $p_{O'scope} = p_{spectrumanalyzer}$, where

$p_{O'scope} = \frac{\left(v_{pk}/\sqrt{2}\right)^2}{50}$ [W], and $p_{spectrumanalyzer} = 10^{-3}\ 10^{\frac{p_{dBm}}{10}}$ [W]. You should carefully confirm and understand these relationships.

What about the case where the frequency spectrum consists of more than one sinusoid, as will always occur with any non-sinusoidal periodic signal, like the earlier sawtooth for example? For these cases, we simply sum the power in each harmonic to find the total average power and this must equal $\frac{v_{rms}^2}{50}$. Consider how we handle this theoretically. Expressing the Fourier series in sine-cosine form, we can write for any periodic voltage waveform:

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega t) + b_n \sin(n\omega t)\right] = C_0 + \sum_{n=1}^{\infty}\left[C_n \cos(n\omega t + \theta_n)\right].$$

Where, by basic trigonometry, the harmonic amplitude and phase angle are given by $C_n = \sqrt{a_n^2 + b_n^2}$ and $\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$.

The right-hand side of $v(t)$ is commonly called the **harmonics form** of the Fourier series, where C_0 is the coefficient of the constant DC term, and is ignored here, since practical spectrum analyzers do not have response down to 0 Hz (however, what do think it's equal to?).

It should be self-evident that the power in the *nth* harmonic is $\frac{\left(C_n/\sqrt{2}\right)^2}{R}$ Watts. So it should be evident that the spectrum analyzer displays $10\log\left(\frac{p_{50\Omega}}{1[mW]}\right)$, where $p_{50\Omega} = \frac{\left(C_n/\sqrt{2}\right)^2}{50}$. Notice especially that the practical spectrum analyzer displays only the amplitude at each harmonic; the phase information is entirely lost.

3.0 Experimental Set Up.

How to setup and use the four basic pieces of test equipment introduced earlier, the scope, signal generator, DVM and spectrum analyzer will be discussed during the lab introduction.

3.1 Experiment.

Assemble the experiment according to the instructions given during the introductory lecture, where the function generator drives the spectrum analyzer directly, but a sample of that same signal is connected to the oscilloscope using its high-impedance probe. The DVM should also be connected in parallel with the spectrum analyzer to accurately measure the AC rms voltage coming from the signal generator.

1. **Simple quantitative comparison of some common waveforms.**
   Investigate and compare the following waveforms in both the time and frequency domains using a time-domain frequency of 100 kHz: sinewave, 50% duty cycle square-wave, sawtooth wave. Verify that power is conserved between the two domains, and that Fourier series representations accurately predict what the spectrum analyzer displays. Also verify that the DVM displays AC rms voltages corresponding to each waveform type. Note the DVM will give a more accurate measurements of rms voltage than can be obtained from the O'scope.

2. *Extra Credit:* **Spectrum of a non-ideal square-wave.**
   Consider the 50% duty cycle square wave you investigated in part 1. How many harmonics are needed to actually represent this waveform? Since the signal generator has constant (but non-zero) rise and fall times, accurately measure these transition times using the 10%-90% points on the observed time-domain waveform. It is these finite transition times that limit the actual harmonic content. Note how many harmonics are present, what the highest harmonic frequency is, and whether they all agree with the amplitudes predicted by analytic Fourier analysis.

Write a brief report summarizing your procedure, data and results.